Sculptural Form finding with bending action

Jens Olsson[1], Mats Ander[2], Al Fisher[3], Chris J K Williams[4]

[1]Chalmers University of Technology, Dept. of Architecture, 412 96 Göteborg, jens.gustav.olsson@gmail.com
[2]Chalmers University of Technology, Dept. of Applied Mechanics, 412 96 Göteborg, mats.ander@chalmers.se
[3]Buro Happold Engineering, London W1T 1PD, al.fisher@burohappold.com,
[4]University of Bath, Department of Architecture and Civil Engineering, Bath BA2 7AY, c.j.k.williams@bath.ac.uk

Abstract

This paper demonstrates a method development for integration of bending action into the form finding process. The aim has been to facilitate the designer with means to compromise between structural efficiency and sculptural freedom for grid shell type of structures.

The analysis is carried out using Dynamic Relaxation (DR) and to achieve bending capability, the base element implementation is a 12 degrees of freedom beam where the DR is solving for both translations and rotations [2]. For purpose of validation, stress based utilization is calculated based on Eurocode 3 equations which are simplified to allow for separation of axial and bending utilization [3]. A stress based sizer is also implemented to enable comparison based on tonnage.

The methods presented are derived from the principles behind stiffness control and force-density control form finding, commonly applied for compression and tension structures respectively [1]. The common denominator being that they are all driven by a form finding load case, where the user can specify various combinations of axial- and bending utilization limits for the elements, to which the form adapts as it tries to find equilibrium of internal and external forces and moments.

None of the methods was found to satisfy all requirements for a useful general purpose shell design tool. The main issue was found to be wrinkling of the initial geometry when the need for drastic change in element length, required for a structurally unsound free form surface changing shape in the form finding process, is being opposed by the relatively high axial stiffness of a beam. This is not usually an issue when form finding with elastic springs that undergo large deformations in the process.

Conclusively there might be merit for a form finding element that allows for large axial deformation to avoid the wrinkling problem but has the bending properties of a beam. From a workflow point of view, the implementation of elements with bending capability was demonstrated to be useful for form exploration, particularly when combined with automatic sizing. The separation of bending and axial utilization was also found useful from a form evaluation point of view.

Introduction

The first part of the paper presents the computational frame work behind the study, starting with choice of solver, definitions and coordinate systems. Thereafter, a vector based beam theory is briefly mentioned, including the calculation of forces and moments in the local coordinate system, the subsequent transformation into the global structure, as well as calculation of translation and rotation of the nodes using dynamic relaxation. Furthermore, a stress based utilization is introduced and the separation of axial and bending utilization is presented. Finally, the bending based form finding methods are presented and discussed.

1.Setup

1.1 Base element

With the ambition of creating a form finding tool for design, real time feedback was considered to be of importance and dynamic relaxation was the natural choice of solver. The node wise iterative scheme does not rely on a matrix formulation, hence will not have to comply with constraints from matrix inversion, arguably making development work more flexible and intuitive, although at the cost of speed. In order to introduce bending elements with the DR solver there was mainly two different approached considered.

In the PhD thesis of Adriaenssens S. [2] a 6 dof element is introduced with the capability of capturing bending for symmetrical cross sections with only translational dof. These elements are shown to accurately model elastic splines with high computational efficiency, however, with the limitation of having to be connected in continuous chords for relative angle calculations. This was found problematic when modelling non-surface like structures (i.e. space trusses) but also in the general case for nodes with odd valance.

However, in the same PhD thesis a nonlinear 12 dof beam element solved with DR is introduced for the purpose of benchmarking. Initially developed by Wakefield D. S. [3] and Ong [4] it has been successfully adopted and used for a number of different structural systems Williams [5] and Wakefield [6]. Due to the 12 dof configuration the element is well compatible with the structural design codes, thus allowing for automatic stress based sizing based on euro code utilisation equations for the purpose of tonnage estimation to trace form progressing (where progress is measured in terms of reduced weight). The 12 dof element is also generic in the sense that it allows for modelling of any type of structural topology, suitable for a design tool. Hence the 12 dof element was adopted as the base element of choice for this study.

1.2 Solver

Dynamic relaxation applied to beam elements, may be a rather unconventional approach to structural analysis, and because of it's importance in the method development in section (2.x.x) a short summary of the method follows here.

For a given structural topology, each node is given a unique coordinate system (CS) with 6 degrees of freedom (DOF), allowing translations and rotations about the x, y, z axes, here referred to as the node CS. Similarly, each beam element is given two CSs, with 6 DOF for each end, here referred to as the beam end CSs. The orientation of each beam CS is such that the Z vector is aligned with a vector P that is spanning between the two ends of the element, and the CS position relative the adjacent node is kept constant (unless releases are introduced). Internal forces and moments in the elements are calculated based on; the relative rotation between the two beam CSs for the twist, the rotation relative to the P vector is used for calculating bending moments, and the elongation/shortening of the member due to bowing and translations of the adjacent nodes gives the axial force. The forces and moments are first calculated in the beam CSs and need to be transformed to the adjacent node CS before they are summed to give out-of-balance force and out-of-balance moment that drive the iterative search for equilibrium, [2] (Ch. 2.3.1).

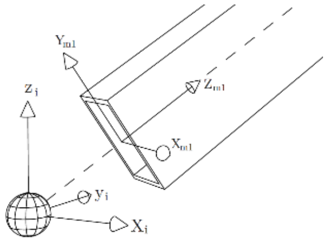

Figure 1. Showing the node coordinate system, and one of the two beam end CSs of an adjacent element m.

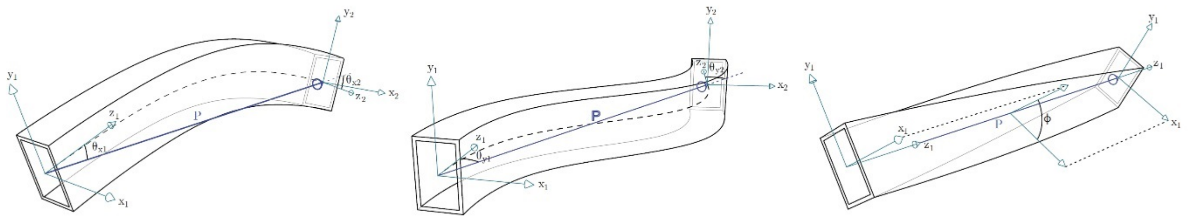

Figure 2. Rotation of the node CS deforms the adjacent element, from left to right rotation around the local x-axis, the local y-axis and the twist measured as the relative rotation of the two end CSs.

The local moments and forces in the elements caused by the movements of the beam end CSs, can be obtained from differentiating the strain energy equation with respect to each deformation mode.

$$U = \frac{EA}{2L_0}(e_a + e_b)^2 + \frac{EI_x}{2}\int k_{xx}^2 dl + \frac{EI_y}{2}\int k_{yy}^2 dl + \frac{GJ}{2L_0}\varphi^2. \tag{1}$$

Where, E is the youngs modulus, G is the shear modulus, I_x and I_y is the second moment of area around the x and y axis respectively, A is the cross section area, k_{xx} and k_{yy} are the curvature around the x and y axis respectively, J is the torsional constant, L_0 is the element length and φ is the twist angle.

1.4 High level algorithm

For a given structural topology including loading, boundary conditions, material properties and section definitions, the following steps are iterated in the DR solver to find the equilibrium state:

While (not converged)

1. Foreach (element in the structure)
    (a). Calculate element forces F_e and element moments M_e based on translations and rotations from previous iteration. *For property control form finding: Scale factors are applied here according to eq. (2) and (3).*

(b). Calculate axial and bending utilisation U_A and U_B respectively, based on section properties and internal moments and forces.

(c). *For limit control form finding: Force and moments are scaled here to fulfil the utilisation limits as described in (2.2.3).*

2. Foreach (node in the structure)

(I). Calculate a fictitious mass m and moment of inertia J for each DOF, tuned for speed.

(II). Iterating through the elements adjacent to the node, transform internal forces and moments from beam end CS to node CS and add on to the node $(M_e \rightarrow M_n,\ F_e \rightarrow F_n)$.

(III). Apply self-weight and super imposed load to the node.

(IV). Move the node in the direction of the out of balance force, $(F_n = ma)$.

(V). Rotate the node in the direction of the out of balance moment, ($M_n = J \propto$).

(VI). Apply the same translations and rotations also to all adjacent beam CSs.

Where F_n is the net force on the node, m is the fictitious mass matrix and J is the moment of inertia matrix both of which tuned for each according to [1], a is the acceleration, M_n is the net moment and $\propto$ is the angular acceleration.

These steps are iterated until the equilibrium positions for the nodes are found and vibrations have died out due to artificial damping. The utilisation for the elements is calculated according to Eurocode 3 specifications for uniform members in bending and axial compression [3] (Ch. 6.3.3). The equations are somewhat simplified, partly to reduce complexity that is not needed in conceptual design, and partly to enable a separation of axial and bending utilization.

There is a last step which is of importance to enable option comparison and progress tracing, and that is automatic stress based sizing. The specific implementation bears resemblance to previous work by [reference?].

2. Bending based form finding

2.1 Form finding

Form finding can be done using many different techniques [4], where arguably the two most common techniques is the stiffness control method which is typically used for compression structures, and the force-density control method which is used for tensile structures. Both methods aim to find the pure axial equilibrium geometry for a structure, but the load case varies and so does the way the forces are derived.

The stiffness method is commonly used for the shaping of compression structures and the process aims to find a structure where the elements are working in pure axial force for a given load case. The structure is usually modelled with spring elements and the form finding load case is often the self-weight and super imposed dead load. The sudden application of load to the initially unstressed geometry puts the nodes out of equilibrium. Using a solver of choice, the nodes are then moved iteratively in the direction of the out-of-balance force, causing the adjacent elements to stretch, introducing a force in the member. The member forces are summed over the nodes, and together with the external load giving the out-of-balance force for the next iteration. The process is iterated until equilibrium between internal and external force is reached.

The Force-density method works in a similar fashion but the external load case is replaced by internal pre-stress, which is determined based on material properties and kept constant throughout the form finding process. Thus, the shape of the structure is the only changing factor for equilibrium to be reached.

2.2 Bending based Form finding

With the aim to develop a method to conduct form finding which allows for a trade-off between initial geometry and fully form found geometry, three different approaches to bending based form finding were explored and are presented below. All of the methods are based on a reversed load case as the governing form driver where the difference is the way in which the force and moments are calculated/set in the elements. Inspiration is taken form the classical techniques that are used for tensile and compression structures presented above.

2.2.1 Property control form finding

This method is implementing a way of controlling axial and bending stiffness for the elements in a structure during the form finding process by scaling the properties, EA and EI, representing axial and bending stiffness respectively. By dropping the bending stiffness to zero the beams are effectively converted into bar elements and the form adapts to find equilibrium without bending. By increasing the bending stiffness again, the form adapts to work partly in bending and partly in axial action where this ratio is controlled by the user. Taking the derivative of the equation (1) with respect to e and introducing an axial stiffness scale factor, s_{axial}, gives the following expression for the axial force in a member;

$$N = \frac{\partial U}{\partial e} = s_{axial} \frac{EA}{L_0} (e_a + e_b) \,. \tag{2}$$

Similarly, by expressing the curvature around the x axis, k_{xx}, in terms of the angles at the end rotations, θ_{x1}, θ_{x2} and by differentiating equation (1) with respect to θ_{x1} [2], and introducing the bending scale factor s_{bend} the moment at end 1 around the x axis can be expressed as;

$$M_{x1} = \frac{\partial U}{\partial \theta_{x1}} = 4 \frac{NL_0}{30} \theta_{x1} - \frac{NL_0}{30} \theta_{x2} + s_{bend} (4 \frac{EI_x}{L_0} \theta_{x1} + 2 \frac{EI_x}{L_0} \theta_{x2}). \tag{3}$$

The same principle applies for moments M_{x2}, M_{y1}, M_{y2} as well as for the twisting moment M_{φ}. Note that even if the bending scale factor is set to zeros, there is still a contribution to the moment from the axial force.

This method was found to work well for the most structures, and it clearly communicates the difference in form between a bending structure and the pure catenary structure.

2.2.2 Force/Moment control form finding

This type of form finding is inspired by the method used for tensile structures. Instead of calculating the axial force and the moments based on translations and rotations, the values are set based on the capacity of the elements. The user controls the force and moments in the elements by specifying the value as a percentage of the axial/bending capacity. In contrast to force density method the governing form driver is a load case, thus, it becomes a tricky balance act to ensure that the forces set in the elements can balance the applied loading. If the load is large compared to the element capacity, equilibrium might never be found since the force does not change with the elongation of the members. Hence, a method that was not found practically useful.

2.2.3 Limit control form finding

Limit control form finding has similarities to the two methods presented above. It allows for elaboration with the ratio of bending and axial action like in property control, but rather than scaling the EA and EI values for the elements the user specifies limit values for the axial and bending utilization in a way inspired by the principle behind force/moment control. These two limits are percentages of the member capacity. If a member is loaded such that the axial force or the moment exceeds the specified limits the exceeded quantities are scaled down and the form will have to adapt. This approach makes it possible to enhance the structural performance in the most critical parts of a structure, where for example the bending action is dominating, without necessarily changing the parts that are not as critical.

Below is a high level algorithm that show how the principles behind the limit control form finding method. For a given limit of axial utilisation a^{lim} and bending utilisation b^{lim}, where S_G and S_M describes the geometric and material properties of the section and R represents a collection of the reduction factors due to flexural buckling and material imperfections, the high level algorithm looks like:

1. Execution of step (a) as described in section 1.4, where the element forces F_e and moments M_e are calculated based on displacement and rotations of the nodes.

2. Execution of step (b) as described in section 1.4, where the axial and bending utilization $U_a(F_e, S_G, S_M, R)$ and $U_b(F_e, S_G, S_M, R)$ are calculated respectively.

3. Calculating the $U_a^{lim} = a^{lim}U_a$, and $U_b^{lim} = b^{lim}U_b$

4. If $U_a > U_a^{lim}$, then scale the force F_e such that $U_a = U_a^{lim}$.

5. if $U_b > U_b^{lim}$, then scale moments M_e such that $U_b = U_b^{lim}$.

6. Execution of step (I) – (VI) continues and each node (and adjacent CS) is translated and rotated as a response to the out of balance force and the out of balance moment.

2.3 Limit Control Form Finding 2D Example

2D Example of a sub optimal arc structure under the influence of a form finding load case. This example clearly shows the how the form is adapting to the given utilization limits and how the combined utilization plot in figure 5 can be used as a means for progress tracing and comparison.

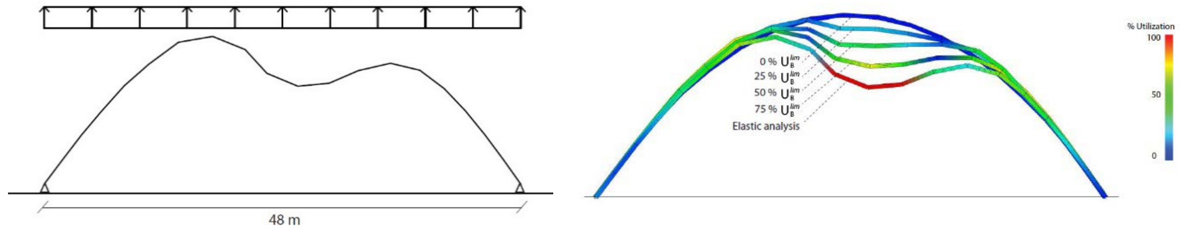

Figure 3. Image to the left show the boundary condition, loading and initial center line geometry for a bent arc like structure. Image to the right show the equilibrium geometry as a result of elastic analysis and 4 cases of limit control form finding with different utilization limits. The colors display the utilization of the member capacity where red I 100%

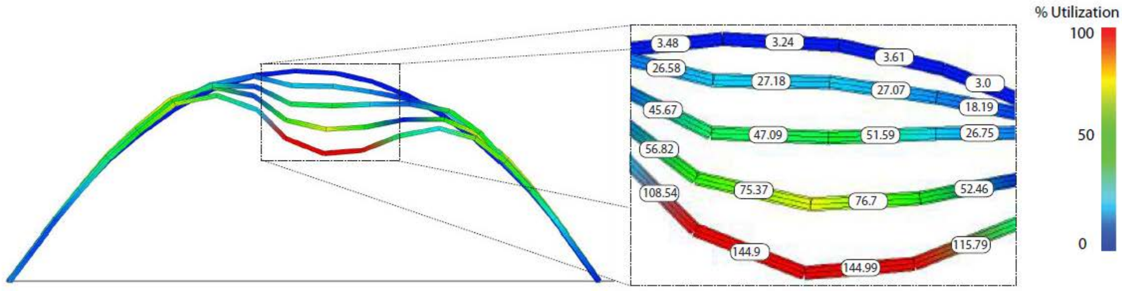

Figure 4. Combined axial and bending utilization for the critical part of the structure can be used as a measurement for efficiency. As expected, the pure catenary structure outperforms the other options by an order of a magnitude.

2.4 Limit Control Form Finding 3D Example

Example of a free from surface structure where limit control form finding is used to improve the shape and automatic sizing is used for evaluation.

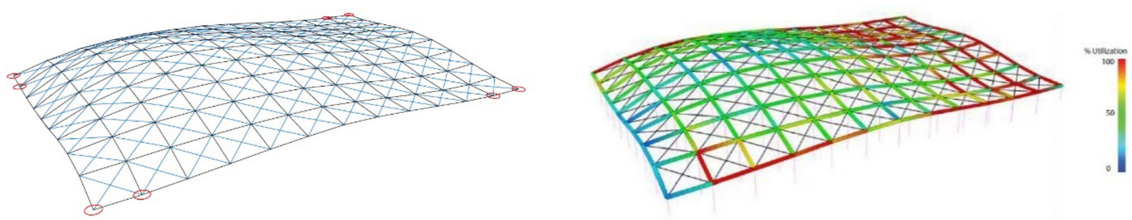

Figure 6. Initial set up of the structure to the left with beam elements is black and cable elements colored blue. The Surface patches indicating the load direction for an even distributed 1 kN/m line load. Image to the right shows the utilization for the initial configuration.

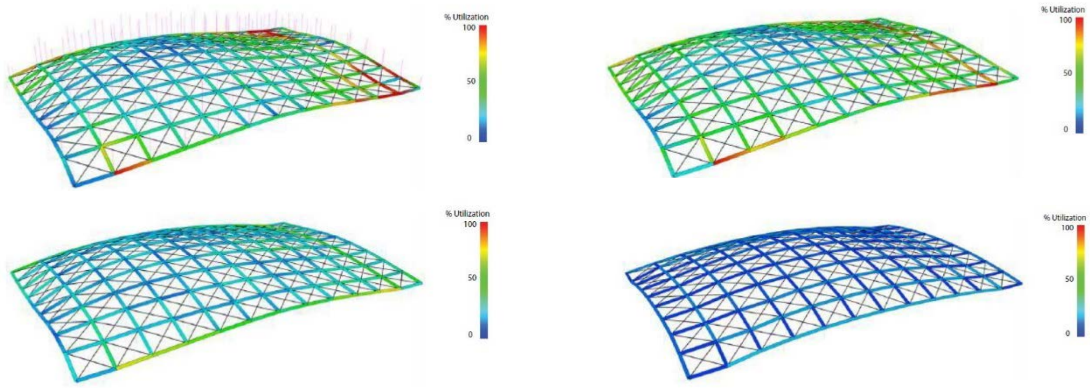

Figure 7. Structure colored by utilization under the influence of a reversed 1 kN/m form finding load case. The top left corner showing the initial geometry, the top right corner showing the structure under 50/50 utilization, the bottom left 80/20 utilization and the bottom right 100/0 axial and bending utilization limits respectively.

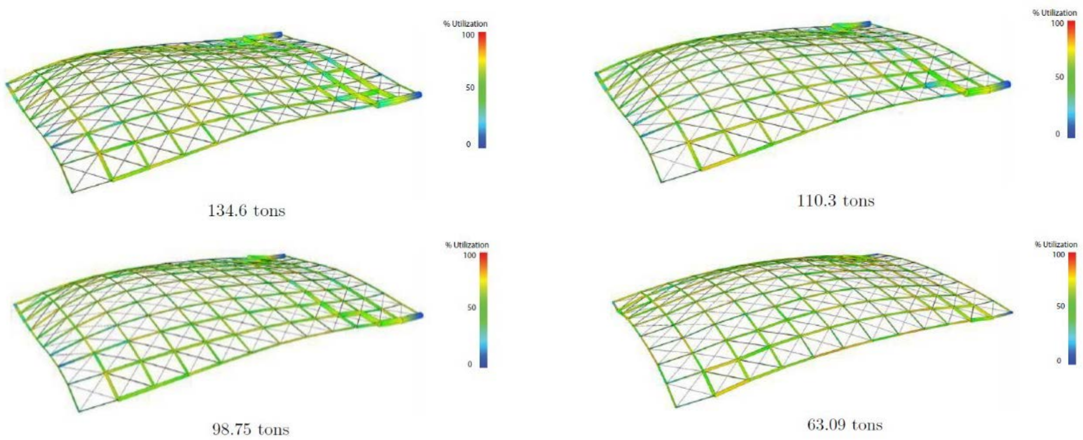

Figure 8. Size optimization results for the 4 options showing reduced weight as the structure functioning more as a shell.

4. Discussion

Among the three different methods that was developed and tested in this study the property control method was found the most stable. It clearly communicates the difference in axial and bending action, but the scale factors are applied independently of utilisation, so the whole structure is treated in the same way, regardless of structural contribution should that be in bending or in axial action. The force/moment control method was not found practically useful, due to the challenge of balancing internal and external forces/moments. The limit control method was found to have the most potential and was working well for the 2D case, but suffering from some fundamental issues in the 3D implementation as the resulting geometry often ends up as non-smooth and wrinkled. A drastic change in shape in the form finding of a surface structure, will most often mean a drastic change in surface area for that structure, thus a drastic change in lengths for the discretised elements that constitutes that structure. The length change is not a problem if the surface is modelled with a network of elastic springs that are allowed to undergo large deformations in the process. It also works well for the 2D case of limit control form finding, but becomes problematic in the 3D implementation. The introduction of beam elements with realistic materials, section properties and utilisation limits, as required to capture bending capability, results in a large axial stiffness constraining the change of length for an element, ultimately leading to wrinkling.

References

[1] Michael R. Barnes. Form Finding and Analysis of Tension Structures by Dynamic Relaxation. International Journal of Space Structures 14 (1998), 89 –104.

[2] Sigrid Maria Louis Adriaenssens. “Stressed spline structures”. PhD thesis. University of Bath,

[3] British Standards Institution. Eurocode 3: Design of steel structures. 2005.

[4] Veenendaal D. Block P. An overview and comparison of structural form finding methods for general networks, International Journal of Solids and Structures 49 (2012) 3741 – 3753.